Design of blade tip timing measurement systems based on uncertainty analysis

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GOAL:

• A new calibration and testing technique for development and performance analysis of these kind of measurement systems

• Key components of a new measurement system, made with new sensing elements and a proper data acquisition and processing hardware

• New mechanical modeling and data processing software/methods.
The BTT measurement principle

The basic idea is that if the blade tip arrive in late is backward deflected, if it arrive earlier is forward deflected

If $\Delta t$ is the delay

deflection $s = v \Delta t = 2\pi R(\Delta t / T)$
A simple description of the measurement technique

For each blade passing on each sensor we have (at least) one displacement sample.
Kind of blade vibrations

Blade vibration frequency normally higher than rotational frequency -> on each turn many vibration cycles

Random vibrations or transient? Difficult to take into account in the design of BTT measurement systems!

- Integral
  - Single mode
  - Multi mode

- Non integral (travelling waves)
  - Single mode
  - Multi mode
Kind of blade vibrations – vs Sampling
where sensors should be installed?

Few sensor -> Aliasing

S(t)

- Integral
  - Non integral (travelling waves)
  - Integral
    - Single mode
    - Multi mode
  - Non integral (travelling waves)
    - Single mode
    - Multi mode
Why using non equally spaced sensor is better?

Non equally spaced sensors allows measurement of both integral and non integral vibrations and more frequencies with a reduced number of sensors.
Not equispaced sampling – sensor optimal positions

2 samples on each period of each vibration frequency components

waveform of blade vibration

\[ \alpha_1 \, \alpha_2 \, \alpha_3 \, \alpha_4 = \text{Sensors positions} \]
Sensor positions -> Mesurable vibration Bandwidth

\[ f_r : \text{revolution} \quad f: \text{vibration frequency} \rightarrow \text{a period of blade vibration occur along an arc of amplitude:} \]

\[ \alpha = \frac{2\pi f_r}{f} \]

to avoid an aliased sampling of the \( s(t) \):

\[ f_{\text{max}} = \frac{2\pi f_r}{\alpha_{\text{min}}} \]

- the angle \( \beta \) inside which all the four sensors are included fix the lowest measurable frequency, we have

\[ f_{\text{min}} = \frac{2\pi f_r}{\beta} \]

\[ \alpha_{\text{min}} - \beta \rightarrow \text{bandwidth} \quad f_{\text{min}} - f_{\text{max}} \]
Displacement sample estimation

\( S_i \) is determined by the \( \Delta t_i \) variation with reference to the mean values \( t_{m,i} \) of the times of transit of the blade in front of the sensors. If the two sensors are spaced between them of an angle \( \theta \):

\[
S_i = V_p \ \Delta t_i = \omega_m R \Delta t_i = \theta R \ \Delta t_i / \Delta t_m
\]

\( R \) is the radius of the rotor; \( \omega_m \) has been estimated by the \( \theta / \Delta t_m \) ratio, defining \( \Delta t_m \) as the mean transit time of the blade from the first to the second sensor.

\[
\Delta t_i = \left[ \Sigma_i (t_i - t_{0,i}) / N \right] - (t_i - t_{0,i})
\]

\[
\Delta t_m = \left[ \Sigma_j (t_j - t_{0,j}) / N \right]
\]

Where \( t_i \) is the transit time on a sensor, \( t_0 \) on the previous sensor and the index \( i \) is referred to the subsequent passings on the two sensors (for example at the \( i \)-th revolution of the rotor). Finally we get:

\[
S_i = \theta R \left[ 1 - N (t_i - t_{0,i}) / (\Sigma_j (t_j - t_{0,j})) \right]
\]
the uncertainty $\delta S_i$ can be estimated equal to the uncertainty of the ratio considering that, if $\delta t$ is a estimate of uncertainty on the measurement of each instant $t_i$, the uncertainty on the numerator of such ratio, that is on the difference $(t_i-t_0,i)$, will be $2\delta t$ at the maximum. The uncertainty on the denominator, equal to the mean value of these differences, can be estimated with the value $2\delta t/N^{1/2}$. The relative uncertainty of a ratio can be overestimated by adding the relative uncertainties of the factors, therefore we have

\[
\delta S_i / S_i \approx 2\delta t / (t_i-t_0,i) + 2\delta t / (\delta t_m N^{1/2})
\]

Considering that $\Delta t_m = \theta / \omega_m \approx (t_i-t_0,i)$ the previous relation becomes:

\[
\delta S_i / S_i \approx (2\delta t \omega_m / \theta)(1+ 1/N^{1/2})
\]
Uncertainty sources

The uncertainty on the $t_j$ depends mainly on:

- Irregularity degree of the rotation which causes variations in the $\Delta t_i$ not connected to the measurand, which can be estimated equal to:

$$\delta t_i = \Delta t_i i = i \frac{T_{\text{medio}} \theta}{2\pi},$$

where $i = (T_{\text{max}} - T_{\text{min}})/T_{\text{medio}}$ is the irregularity degree of the rotation and $T_{\text{max}}$, $T_{\text{min}}$ and $T_{\text{medio}}$ are respectively the maximum, minimum and mean rotation periods.

When $i = 0.01\%$ at 11000 rpm, with 4 sensors and therefore $\theta/2\pi = 1/4$, we have $\delta t_i = 140$ ns

- Uncertainty of the time base of the sampler: typical values of about $\delta t_{\text{III}} = 2$ ns and therefore negligible.

- Uncertainties connected to the combination of noise and rise time of the sensor signal
Sensor performances: noise-rise time combination

- Bandwidth
- Rise time (little sensitive area)
- Signal to noise ratio

Rise time $R_t$ strongly affect timing uncertainty

$$R_t = \left( \frac{0.35}{f_{3dB}} \right) + \left( \frac{d}{v} \right)^2$$

Rise time due to blade cutting at velocity $v$ of sensitive area of length $d$
Uncertainty due to rise time – noise combination

\[ \frac{N}{U_t} = \frac{\Delta V}{R_i} \]

\[ \text{STR} = \frac{\Delta V}{N} \]

\[ U_t = \frac{1}{\text{STR}} \sqrt{\left(\frac{0.35}{f_{3dB}}\right)^2 + \left(\frac{d}{v}\right)^2} \]

\[ \delta S_i = \omega R U_t \]

Uncertainty in timing depends on sensor signal to noise STR ratio and on rise time.
Problem: How to connect the different noise sources?

- dS due to sensor rise time and noise
- Average pulse timing positions
- Irregularity degree of rotation
- Sampling time

\[ R_t = \sqrt{(R_1^2 + R_2^2 + \ldots \ldots + R_i^2) \right] } \]
Uncertainty experimental qualification: proposal of a new calibration technique

Motion decomposition: rigid blades, vibrating sensors
A better solution

Blade simulated by rigid tooth at sensor positions on the rotor
Conclusion

• BTTMON research project started

• Performend uncertainty analysis, useful to define a knowledge base for proper design of this kind of measurement systems

• Proposal of a new technique for calibration of BTT measurement systems